The Effectiveness of Asset, Liability, and Equity Hedging Against the Catastrophe Risk: The Cases of Winter Storms in North America and Europe

Yang-Che Wu [a], Ming Jing Yang [a], Jo-Yu Wang [b], and Wen-Lin Wu [b]

[a] Department and Graduate Institute of Finance, College of Finance,
Feng Chia University, Taichung 40724, Taiwan
E-mail addresses: wuyangche@fcu.edu.tw (Y.-C. Wu), mjyang@fcu.edu.tw (M. J. Yang)

[b] Department of International Trade, College of Business,
Feng Chia University, Taichung 40724, Taiwan
E-mail addresses: jywang@fcu.edu.tw (J.-Y. Wang), wlwu@fcu.edu.tw (W.-L. Wu)

Abstract

The winter storms in North America and Europe are responsible for the majority of the natural catastrophe insured losses. This study analyzes the effectiveness of insurers hedging against the winter storm risk in terms of asset (catastrophe derivatives), liability (catastrophe bonds), and equity (catastrophe equity puts) risk management perspectives. The corresponding accounting procedures are designed for the insurers with different hedging strategies. The cash flows are simulated according to these accounting procedures under different loss models. The findings of our study indicate that the Cox-Ingersoll-Ross model (Cox et al., 1985) is the better-fitting model for the insured storm losses in North America and Europe. The numerical analysis results of the financial performance show that our suggested hedging strategies are effective based on the long-term positive profit and the improvement in the insolvency ratios. The insurance premiums for different terms are analyzed to find the appropriate term with the less volatile premiums. The conclusions of this study provide the insurers with a more diversified portfolio under the catastrophe risk management.

Keywords: Catastrophe Derivatives, Catastrophe Bonds, Catastrophe Equity Puts, Catastrophe Risk Management

***JEL classification*:** G32; G11; C15

1. Introduction

Severe convective storms can produce strong tornados, large hails, heavy snowfalls, thunders and lightings, icy winds, and flash floods. According to the Sigma data from Swiss Re 2015, the insured losses from the global natural catastrophes and the severe convective storms between 1990 and 2014 grew at the average annual rates of 7.7% and 9.0%, respectively. As shown in Figure 1, the proportion of the insured losses from these storms is estimated at over 50% of the total natural catastrophe insured losses for the period of 2000-2014. In 2014, catastrophe-related losses in North America were primarily caused by the convective winter storms. In Europe, the insured losses totaling USD 6.6 billion in 2014 primarily also resulted from the convective storms and heavy precipitation.

[Insert Figure 1 here]

Reinsurance is a traditional form of asset hedging for the insurers. However, its use involves significant transaction costs, moral hazard, and credit risk. When a particularly large catastrophic event occurs, these risks become more serious. Corporate demands for more effective hedging strategies are not satisfied with the traditional insurance policies due to the insurance capacity constraints in the insurance and reinsurance markets. The catastrophe derivatives in the capital market are written on the loss-related indexes or triggers tied to some indexes such as the Property Claim Services (PCS) Catastrophe Loss Indexes. These types of derivatives can be used to reduce transaction costs, moral hazard, and credit risk significantly, but will incur basis risk instead. Because the weather-related events occur frequently and severely, a broad range of weather derivatives (futures, options, and swaps) are available currently. These products are based on some natural indicators (temperature, rain, snowfall, wind, and frost) such that market participants can manage the global weather-related risks, including catastrophic hurricanes, heat waves, and cold breaks. The appropriate risk management by using the weather derivatives will increase energy firms' value (Perez-Gonzalez and Yun, 2013). This study further shows that these weather derivatives will also benefit the insurers.

Because catastrophic events lead to large claims being listed on the liability of the insurer's balance sheet, the liability hedging would be the most direct means to control the risks. Catastrophe bonds (CatBonds) are a popular instrument for liability-linked securities. The bondholders pay the special purpose vehicle some principal amount to compensate for the claim payments under some predetermined conditions after the insurers promise to share certain premiums (namely a "high-yield rate"). The credit risk from the insurers can be avoided because of the use of a special purpose vehicle. However, the issue costs of CatBonds depend heavily on the insurer's credit rating. The CatBonds originated in the mid-1990s when the Hurricane Andrew and the Northridge earthquake in California caused the insured losses of approximately USD 30 billion. Except for the period during the global financial crisis in 2008, the historical growth rate of the CatBond market has been around 20% annually since they were first issued.

Issuing new shares in the capital market is a common way for companies to raise funds. Similarly, the insurers can design a mechanism of contingent capital to exchange their equity or contingent surplus notes for funds under some predetermined conditions for catastrophic events. Some literature has gradually focused on the catastrophe equity puts (CatEPuts) since the contingent surplus note is actually a special case of CatEPuts. CatEPuts are a type of customized product in which credit risk and moral hazard exist. The insurer needs to find a counterparty and then negotiate a committed fee. The writers of CatEPuts are almost always large global reinsurance companies such as Centre Re and Swiss Re. Since RLI Corporation issued the first CatEPut in October 1996, three CatEPuts have been exercised among the seven CatEPuts that have been written (two from Centre Re, four from Swiss Re, and one from UBS) as of 2010.

The natural catastrophe (NatCat) loss model determines how effectively the asset, liability, and equity hedging against catastrophe risk performs. Most of the literature develops a NatCat loss model by adopting a pure Poisson process for the event occurrence and by taking a positive random variable to describe the event impact. Lin et al. (2009) propose a doubly stochastic Poisson process to model the arrival process for catastrophic events. Wu and Chung (2010) improve the pure Poisson process by using a cyclical trend arrival rate that captures the occurrence of catastrophic events better than the constant arrival rate and the regime-switching arrival rate. The alternative log-loss jump size follows the exponential (Christensen, 1999), normal (Vaugirard, 2003), lognormal (Burnecki et al., 2000), gamma (Wang and Jaimungal, 2006), Pareto (Powers et al., 2012), and generalized extreme value distributions (Abdessalem and Ohnishi, 2014). However, these stochastic processes assume that the occurrence intensity and the jump size distribution are independent. Smith and Shively (1995) use a point process to calibrate the relationship between the frequency and size of loss data. The previous studies fit the distributions to the catastrophe losses in general rather than the insured losses of some specific events. Because different types of NatCats will result in different levels of property loss and victims, this study presents a loss model for the insured loss specifically caused by the winter storms in North America and Europe.

There are three major contributions of the study. First, as winter storms have caused most NatCat-related insured losses in North America and Europe, our empirical study is based on the winter storm losses and finds that the Cox-Ingersoll-Ross (CIR) model is the best-fitting model for the insured storm losses in these areas. Second, since the catastrophe losses are significantly different across time, the insurance premiums become highly volatile. We present two feasible methods of premium payment that can be used to reduce the volatility of the annual insurance premium. One method is for the insured to purchase the n-year insurance policy from the insurer, but the premium is amortized within n years, i.e., the annual premium is the same during the n years. The premium will be adjusted every n years. The other method is that the insured can also buy the n-year insurance policy with only $1/n$ share of the original maximum claim each year. Then, the insurance coverage will be almost invariable because there are always n policies to cover the NatCat risk each year. The numerical analysis results for various insurance periods show that the coverage term of $n = 3$ makes the premium less volatile. Third, we further develop the accounting procedures for the asset, liability, and equity hedging and estimate the financial performance in different scenarios. The numerical analysis results show that our suggested hedging methods are effective owing to the long-run positive profit and the improvement in insolvency ratios. The conclusions of this study provide the insurers with a more diversified portfolio under catastrophe risk management.

The paper is organized as follows. Section 2 establishes the catastrophe loss model for the risk-adjusted capital and introduces the technical insurance premium formula. Section 3 develops the accounting procedures for the liability, equity, and asset hedging. Section 4 calibrates the loss model. Section 5 conducts the numerical simulations and analyzes the results. The conclusions are provided in Section 6.

2. The catastrophe loss model and the insurance premium formula

2.1 The insurance loss model

The insurer must possess enough reserves for catastrophic event policies. The amount of reserves depends on the NatCat risk. Greater reserves offer more protection against the NatCat risk, but the potential capital cost is also greater. The reserve is a type of risk-adjusted capital. The risk-adjusted capital RAC with a coverage rate of θ offers the risk cover ratio θ of NatCat loss. This study assumes that the risk-adjusted capital is prepared for a 1-in-100-year catastrophic event. As NatCat losses show a serially dependent trend (Wu, 2015), we assume that the NatCat loss process follows a time series of $\{X(t)\}_{t=0,1,2,...}$. The corresponding risk-adjusted capital is denoted by $RAC(t)$, which is defined as

$$RAC(t) \equiv F_{X(t)}^{-1}(\theta), \tag{2.1}$$

where $F_{X(t)}^{-1}$ is the inverse of the quantile θ of the cumulative distribution function $X(t)$.

2.2 Insurance premiums and business cycles

Assume reinsurers in the global insurance market are willing to insure for the NatCat risk whenever possible, i.e., they would resign only when they enter bankruptcy. All reinsurers are assumed to have the same initial capital and cost of raising capital, require the same expenses and risk premiums, and use the same formula for insurance premium calculations. The technical premium $TP(t)$ for year t should cover the expected insured loss plus the risk premium plus the internal expense and operational cost. In addition, the technical premium should be adjusted to reflect the real claim. We also assume that the expected claim is determined by the NatCat distribution and is then adjusted by the loss ratio. The NatCat distribution is formed according to the relative long-term insured loss data. The loss ratio is calculated by the NatCat claims in recent years. In practice, 63% of the past claims are, on average, paid within a year and 82% within two years (Von Dahlen and Von, 2012). Therefore, the loss ratio for the year t is defined as follows:

$$LR(t) = \frac{0.63X(t) + 0.19X(t-1) + 0.18X(t-2)}{TP(t)}. \tag{2.2}$$

The practical premiums governed by the practical (re)insurance market seldom stay at a certain level. The premiums rise quickly when a NatCat occurs and stay at a high level in the subsequent years. At this time, the shareholders of the insurance companies would require more risk premiums (higher cost of capital rate), which the insured are also more willing to accept. This is the so-called hard market. A succession of NatCat disasters would bring heavy claim losses for the insurers and thus encourage a rise in the premium rate. However, as the years pass after the NatCat disaster, the premiums would gradually fall (lower cost of capital rate) because the insurers would obtain a greater profit in good years, and the experience of suffering from the disasters recedes gradually. The market would then enter a soft period. However, the premiums will still stay above the least technical premium for satisfying the minimum rate of return required by the shareholders.

The risk premium required by the shareholders is regarded as the cost of capital, $\kappa \cdot RAC(t)$, where κ denotes the cost of capital rate and can be interpreted as the risk premium rate required by the shareholders to invest in the insurance company. The internal expense and operational cost are denoted by $e(t)$. Most insurance policies have a maximum limit $M(t)$ of claims to restrict the insurer's liability. As the limit $M(t)$ becomes larger, the insured obtains more protection, and thus, a higher premium is charged; however, $M(t)$ must be less than $RAC(t)$ to ensure that the insurer can pay the full claim.

Therefore, the technical premium $TP(t)$ (collected at time $t-1$) is defined as follows:

$$TP(t) = \frac{E^{M(t)}[X(t)] \cdot LR(t-1) + \kappa \cdot RAC(t) + e(t)}{1+r}, \tag{2.3}$$

where $E^{M(t)}[X(t)] = \int_0^{M(t)} x \cdot f(x)dx$, $f(x)$ is the probability density of $X(t)$. r is the risk-free interest rate.

Since the occurrence of and damage from NatCats are irregular, the premiums will also be volatile each year according to Eq. (2.3). The unstable premiums cause an uncertainty of supply and demand for NatCat insurance policies. The insurer would not have a stable expected premium income to make a proper capital planning. The insured would also be concerned about the budget for insurance. This phenomenon would damage the insurer's financial stability and discourage the insured from NatCat insurance policies, thus affecting insurance market operations in the long term.

The insured can take out the multi-year insurance or a one-year policy in a rolling year to stabilize the NatCat premiums. The duration of property insurance is short, usually one year, due to the more irregular events in property insurance than those in life insurance. However, the insured may negotiate with the insurer to write the several-year insurance policy and amortize the premium per year. If the duration is assumed to be n years, the loss ratio at the end of year t is computed by accounting for the claims and premiums in the previous n years:

$$LR(t) = \frac{\sum_{i=0}^{n-1} X(t-i)}{\sum_{i=0}^{n-1} TP(t-i)}. \tag{2.4}$$

The loss ratio is then used to calculate the insurance premium for the next n-year duration. The insurer must also estimate the claims and the risk-adjusted capital in the future n years. The annual premium is estimated by amortizing the n-year premium by a discount of the risk-free interest rate as follows:

$$TP_n(t) = \frac{\sum_{i=0}^{n-1} E^{M(t+i)}[X(t+i)] \cdot LR(t-1) + \kappa \sum_{i=0}^{n-1} RAC(t+i) + e(t)}{\sum_{i=0}^{n-1} (1+r)^i}, \tag{2.5}$$

where $TP_n(t)$ denotes the future annual insurance premium with yearly maximum claim $M(t+i)$ for the n-year insurance duration. Because the catastrophic events occur randomly and cause unpredictable damage, the duration n the insurer allows is shorter than that of life insurance. The stable premium is at least advantageous to both the insurer and the insured for several years. The premium would be adjusted every n years.

Moreover, the insured can also take out the insurance policy of n-year duration but with only $1/n$ share of the original maximum claim each year, $M(t+i)/n$. We call it the rolling-year insurance. That is, the insurance coverage can be decomposed into n insurance contracts, where each lasts n years and covers $1/n$ share of the maximum claim. For example, if this year is t, a contract updated n-1 years ago covers the interval of n years from year t–(n-1) to year t. Then, the insured in year t has n policies ($TP_n(t-(n-1)),\ldots,TP_n(t)$) against the NatCat risk, and the coverage is $\sum_{i=0}^{n-1} M(t-i)/n$, which may approximate $M(t)$. The premium is calculated as follows:

$$TP_n^r(t) = \frac{\frac{1}{n}\sum_{i=0}^{n-1} E^{M(t+i)}[X(t+i)]\cdot LR(t-1) + \kappa\frac{1}{n}\sum_{i=0}^{n-1} RAC(t+i) + \frac{1}{n}\sum_{i=0}^{n-1} e(t+i)}{(1+r)^n}. \quad (2.6)$$

This method allows the insurer to adjust the premium every year using the new catastrophe information, but the insured only bears $1/n$ of the premium due to $1/n$ of the maximum claim, $M(t+i)/n$.

3. The accounting procedures for liability, equity, and asset hedging

A catastrophe claim increases the insurer's liability, but the premium income increases the insurer's assets. The insurer's equity decreases if the claim exceeds the premium. Except for the maximum claim, once the equity becomes negative, the insurer will face default. Therefore, the methods for reducing liability or enlarging equity or assets will effectively hedge the NatCat risk when a catastrophic event occurs. For instance, the catastrophe bonds are a popular instrument of liability securitization to pay the claims of bondholders. Besides, the insurer can also exchange its equity for the pre-specified capital by exercising the catastrophe equity put option when the heavy catastrophe loss increases and its share price falls to the predetermined triggers. In addition, the insurer can also use the weather derivatives to offer a contingent capital to bolster its assets against the unexpected major catastrophic events. The insurer may pay some derivative premiums in good years to obtain some profits for the claims in bad years.

3.1 Basic accounting procedures to measure the profit and loss

The insurer has initial assets $RAC(1)$ for issuing NatCat insurance policies in year 1 and then continues to prepare $RAC(t)$ at the end of year t-1 for issuing policies in year t. Over the period of the entire business, the insurer is assumed to operate at its best because the maximum limit of claims prevents the occurrence of default. When its operation results in profits, debts have priority for repayment, and the remainder is paid as dividends. The insurer can further increase some liabilities as long as it can still match the least requirement of capital for new policies in the next year.

The new capital for the next year is also set at the end of the year t when the actual amount of the aggregate claim payment is known. The underwriting result $UR(t)$ of the insurer at the end of year t is the result of premium income minus claims and expenses, which is calculated by the following equation:

$$UR(t) = TP(t) - \min\left[X(t),\ M(t)\right] - e(t), \tag{3.1}$$

where $M(t)$ denotes the design of the maximum claim. Moreover, the insurer has two additional incomes: the interest earned on the previous-year capital and premiums, and the adjustment of the risk-adjusted capital between the current year and the subsequent year. Therefore, the operating result at the end of year t is

$$OR(t) = UR(t) + r \cdot (RAC(t) + TP(t)) + RAC(t-1) - RAC(t), \tag{3.2}$$

where r is the risk-free interest rate.

We assume that the insurer contracts a loan to satisfy the capital shortfall. The liability in this study only covers operational costs and excludes other purposes, such as investments. The accumulated liability $L(t)$ by the end of year t involves the previous-year liability $L(t-1)$, the adjustment ($RAC(t+1)-RAC(t)$) of the risk-adjusted capital for polices in the subsequent year, and the operating result $OR(t)$. It is modeled by:

$$L(t)=\max\left[L(t-1)+RAC(t+1)-RAC(t)-OR(t),\ 0\right]. \tag{3.3}$$

If the accumulated liability drops, it means that the positive operating result has paid some liability. The adjusted operating result $AOR(t)$ at the end of year t is estimated by:

$$AOR(t)=OR(t)-\max\left[L(t-1)-L(t),\ 0\right]. \tag{3.4}$$

Therefore, the profit before taxes $PBT(t)$ is computed by $AOR(t)$ subtracted by the interest of previous-year liability $cL(t-1)$, which is expressed by:

$$PBT(t)=AOR(t)-cL(t-1), \tag{3.5}$$

where c is the loan rate for liability. If the profit before taxes $PBT(t)$ is negative, the government provides a tax shield with rate γ to encourage the NatCat market by increasing future capital. The accumulated amount of deferred taxes $DTAX(t)$ until year t is calculated by:

$$DTAX(t)=\max\left[DTAX(t-1)-\gamma\cdot PBT(t),\ 0\right]. \tag{3.6}$$

If the profit before taxes $PBT(t)$ is positive, the taxable amount is $PBT(t)$ subtracted by $DTAX(t-1)$. The tax payment $TAX(t)$ is charged with the tax rate τ:

$$TAX(t)=\tau\max\left[PBT(t)-DTAX(t-1),\ 0\right]. \tag{3.7}$$

The profit after taxes $PAT(t)$ at the end of year t is thus computed by the profit before taxes minus the tax payment:

$$PAT(t) = PBT(t) - TAX(t) \tag{3.8}$$

From the perspective of shareholders, the shareholders' equity should evolve in line with the insurer's operational performance. This study has simplified the accounting procedures. The insurer's asset only includes the operational necessity, i.e., the risk-adjusted capital. The profit after tax $PAT(t)$ is regarded as the dividend paid to the shareholders. The equity $E(t)$ at the end of year *t* equals the necessary capital $RAC(t+1)$ next year minus the necessary liability $L(t)$:

$$E(t) = RAC(t+1) - L(t). \tag{3.9}$$

The accumulated dividends from year 0 to *t* are

$$D(t) = (1+r)D(t-1) + \max\left[PAT(t),\ 0\right], \tag{3.10}$$

where $D(0)$ is assumed to be 0. The annual profit $AP(t)$ to the shareholders includes the interest on the previous dividend payment plus the increase in dividend from profit after tax plus the increase in equity (from the possible reduction in liability), which is modeled by

$$AP(t) = rD(t-1) + PAT(t) + (E(t) - E(t-1)). \tag{3.11}$$

Shareholders' wealth by the end of year *t* is the sum of the accumulated dividends in their account and the final equity $E(t)$ of the insurance company:

$$W(t) = D(t) + E(t). \tag{3.12}$$

Of course, the shareholders hope that the insurer can maximize their wealth. However, the insurer must balance profits and insolvency risk because default would severely impact shareholders.

3.2 The liability hedging: catastrophe bonds

Consider that an insurer issues a CatBond to hedge its catastrophe risk. The contract is designed as 1-year zero-coupon bond with a discount factor of ρ. Bondholders pay the amount $\rho N(t)$ to the special purpose vehicle (SPV) at the beginning of year *t* and receive the payoff $PO_{bond}(t)$ at the end of year *t* from SPV:

$$PO_{bond}(t) = \max\left[N(t) - \max\left[X(t) - (M(t) - N(t)),\ 0\right],\ 0\right], \qquad (3.13)$$

in which bondholders offer the additional funds of $N(t)$ to share the claim from $M(t) - N(t)$ to $M(t)$. We assume that $N(t)$ is based on the predetermined ratio *h* of $RAC(t)$. Thus, the technical premium $TP_{bond}(t)$ for the CatBond is adjusted as in Eq. (2.3).

The insurer needs to pay the SPV some interest difference, $id(t)$, between the discount factors of ρ and the risk-free interest rate *r* to guarantee the bondholder's risk premium. Therefore, the underwriting result $UR_{bond}(t)$ for the CatBond issued by the insurer at the end of year *t* is calculated by

$$UR_{bond}(t) = TP(t) - \min\left[X(t),\ M(t) - N(t)\right] - id(t) - e(t), \qquad (3.14)$$

Where $id(t) = \dfrac{N(t)}{1+r} - \rho N(t).$ The maximum of the claim paid by the insurer is $M(t) - N(t)$. Thus, the risk-adjusted capital in year *t* is adjusted to $RAC(t) - \rho N(t)$, and the operating result at the end of year *t* is adjusted by

$$OR(t) = UR(t) + r(RAC(t) - \rho N(t) + TP(t)) + RAC(t-1) - RAC(t), \qquad (3.15)$$

where *r* is the risk-free interest rate. The remaining accounting procedures for the insurer are the same as those in Section 3.1.

From the perspective of the bondholder, he/she can obtain the profit $N(t)(1-\rho)$ in a regular year when $X(t) \le M(t) - N(t)$, but may suffer losses in a bad year when $X(t) > M(t) - N(t)$. The annual profit $AP_{bond}(t)$ paid to the bondholder is calculated by

$$AP_{bond}(t) = rAP_{bond}(t-1) + N(t)(1-\rho) - \min\left[N(t),\ \max\left[X(t) - (M(t) - N(t)),\ 0\right]\right], \qquad (3.16)$$

3.3 The equity hedging: catastrophe equity puts

Assume that an insurer currently has n_1 shares outstanding and purchases a CatEPut from a financial institution at the beginning of each year to hedge its 1-year catastrophe risk. If the CatEPuts are exercised during the period from 0 to *t*, the number of shares outstanding will increase to $n(t)$, and the additional shares issued will be held by the seller of the CatEPuts, such as financial institutions or large reinsurers. The share price in year *t* can be valued as follows:

$$S(t) = \max\left[\frac{E(t)}{n(t)},\ 0\right]. \tag{3.17}$$

Assume that the CatEPut contract is written at the end of year $t-1$. The seller receives the CatEPut premium from the insurer and is required to buy n_2 shares with the strike price of $K(t)$ if $S^*(t)$ is less than $K(t)$ and $X(t)$ exceeds $M(t)-n_2K(t)$. The payoffs of the CatEPut without counterparty risk are designated by

$$PO_{CatEPut}(t) = \begin{cases} n_2(K(t)-S^*(t)), & \text{if } S^*(t) < K(t) \text{ and } X(t) > M(t)-n_2K(t) \\ 0, & \text{otherwise} \end{cases}, \tag{3.18}$$

where $K(t)$ is determined by $\dfrac{h \cdot RAC(t)}{n_2}$. The post-exercise share price $S^*(t)$ is computed by $\dfrac{E(t)+n_2K(t)}{n(t)}$ when the insurer is assumed to issue new shares n_2 for the exercise of the CatEPuts. Thus, the CatEPut can be valued as follows:

$$P_{CatEPut}(t) = E\left[e^{-r}PO_{CatEPut}(t)\right]. \tag{3.19}$$

From the perspective of a financial institution serving as a new shareholder, it receives the premium of the CatEPut from the insurer and needs to infuse new capital into the insurer by purchasing the insurer's new equity in a bad year. We assume that the financial institutional investor owns $n(t)-n_1$ shares in year t. The annual profit $AP_{CatEPut}(t)$ paid to the financial institutional investor is given by

$$\begin{aligned} AP_{CatEPut}(t) = P_{CatEPut}(t) - n_2(K(t)-S^*(t))I_{\left[S^*(t) < K(t) \text{ and } X(t) > M(t) - n_2K(t)\right]} \\ + n(t-1)(S(t)-S(t-1)), \end{aligned} \tag{3.20}$$

When the market becomes more stable and the insurer's stock price is higher than that during its financial distress, the financial institutional investor would start selling these shares in the capital market and get back the funds. This study assumes that the investor sells n_2 shares in the year when the stock price rises above the strike price K. The accumulated wealth of the investor at the end of year t is calculated by

$$\begin{aligned} & W_{CatEPut}(t) \\ & = \sum_{i=0}^{t} e^{ri} P_{CatEPut}(t) - n_2 \sum_{i=0}^{t} K(i) e^{r(t-i)} I_{\left[S^*(i) < K(i) \text{ and } X(i) > M(i) - n_2K(i)\right]} \\ & \quad + \sum_{i=0}^{t} e^{r(t-i)} S(i) n_2 I_{[S(i) \geq K(i)]} I_{\left[\sum_{u=0}^{i} I_{\left[S^*(u) < K(u) \text{ and } X(u) > M(u) - n_2K(u)\right]} > \sum_{u=0}^{i} I_{[S(u) \geq K(u)]}\right]} \\ & \quad + S(t) n_2 \max\left[\sum_{i=0}^{t} I_{\left[S^*(i) < K(i) \text{ and } X(i) > M(i) - n_2K(i)\right]} - \sum_{i=0}^{t} I_{[S(i) \geq K(i)]},\ 0 \right], \end{aligned} \tag{3.21}$$

in which the first term on the right-hand side of the equation denotes the accumulated premium of the CatEPut, the second term represents the outflow of capital from the investor to the insurer due to the insurer exercising the CatEPut, the third term stands for the inflow of capital to the investor if the stock price rises above $K(t)$, and the forth term indicates the market value of the investor's shares.

As the CatEPut brings the additional standby capital $n_2K(t)$, the insurer only needs to provide the capital of $RAC(t) - n_2K(t)$. To be consistent with the previous analysis of the CatBonds, we assume $n_2K(t)$ is equal to $N(t)$. The underwriting result $UR(t)$ of the insurer at the end of year t must include the additional expense of the CatEPut premium.

$$UR_{CatEPut}(t) = TP(t) - \min\left[X(t),\ M(t) - N(t)\right] - P_{CatEPut}(t) - e(t). \tag{3.22}$$

From the perspective of the original shareholder with n_1 shares, he/she owns some proportion of the total equity and should obtain the same proportion of dividends. The accumulated dividends from year 0 to *t* are

$$D(t) = (1+r)D(t-1) + \frac{n_1}{n(t)} \max\left[PAT(t),\ 0\right], \tag{3.23}$$

The annual profit $AP(t)$ for the shareholder is calculated by

$$AP(t) = rD(t-1) + \frac{n_1}{n(t)} PAT(t) + \frac{n_1}{n(t)} (E(t) - E(t-1)), \tag{3.24}$$

and the accumulated wealth of the shareholder from year 0 to the end of year *t* is

$$W(t) = D(t) + \frac{n_1}{n(t)} E(t), \tag{3.25}$$

The remaining accounting procedures for the insurer are the same as those in Section 3.1.

3.4 The asset hedging: catastrophe derivatives

The weather derivatives with an underlying index based on snowfall can be used to hedge the NatCat risk caused by storms, but it may result in a basis risk. For simplicity, this study assumes that the insurer uses the bull spread of call options on a loss index to cover the amount of $N(t)$ for a claim. The spread premium of the derivatives in year *t* thus is the difference between the payoffs of two call options discounted by a risk-free interest rate.

$$d(t) = \mathrm{e}^{-r} E[X(t) - (M(t) - N(t))] - e^{-r} E[X(t) - M(t)], \tag{3.26}$$

where $N(t)$ is the proportional ratio *h* of $RAC(t)$. The insurer (assumed to be risk-neutral) purchases one call option with a lower strike price of $M(t) - N(t)$ and sells another call option with a higher strike price of $M(t)$. Therefore, the underwriting result $UR_{derivative}(t)$ of the insurer at the end of year *t* is calculated by

$$UR_{derivative}(t) = TP(t) - \min\left[X(t),\ M(t) - N(t)\right] - d(t) - e(t), \tag{3.27}$$

where the maximum of the claim paid by the insurer is $M(t)-N(t)$. The remaining accounting procedures for the insurer are the same as those in Section 3.1.

3.5 Performance measures

To analyze whether the catastrophe derivatives, CatBonds, and CatEPuts can be used to reduce the insurers' exposure to natural catastrophe risk and assist the insurers in improving their financial performance, we need to set up some criteria to measure the performance in the four cases presented in Sections 3.1, 3.2, 3.3, and 3.4. The following four criteria, profitability index, modified internal rate of return, Sharpe ratio, and insolvency ratio, are used in our analysis.

3.5.1 Profitability index

One of the most frequently used investment evaluation criteria is the net present value (*NPV*), which is the discounted value of the future cash flows $Z(t)$ of an investment minus its initial cost. In view of the initial capital requirement, a feasible method is the profitability index (*PI*), which quantifies how much an investment contributes to investors' wealth per unit of investment. A value greater than 0 means that the investment is profitable for the investor. The profitability index thus is defined as

$$PI = \frac{NPV}{RAC(1)}, \tag{3.28}$$

where NPV is expressed by

$$NPV = \sum_{t=1}^{T} \frac{Z(t)}{(1+r)^t} - RAC(1), \tag{3.29}$$

where $Z(t)$ is the annual profit $AP(t)$ to the shareholders of the insurer under different hedging strategies discussed above, and T means the life of the investment.

3.5.2 Modified internal rate of return

Another common concept in measuring the performance of an investment is the internal rate of return on the investment. An investment's internal rate of return (*IRR*) is the discount rate that makes the net present value (*NPV*) of the future cash flows $Z(t)$ of an investment equal to zero. The *IRR* is defined as

$$\sum_{t=1}^{T} \frac{Z(t)}{(1+IRR)^t} - RAC(1) = 0. \tag{3.30}$$

A problem with the *IRR* is that an investment may have more than one *IRR* because a cash outflow of the insured losses occurs sometime after the inflows have begun. A modified internal rate of return (*MIRR*) is the discount rate that makes the present value of the future value of cash inflows ($Z(t) > 0$) equal to the present value of cash outflows ($Z(t) < 0$). The *MIRR* is defined as

$$RAC(1) + \sum_{t=1}^{T} \frac{\max\left[-Z(t),\ 0\right]}{(1+r)^t} = \sum_{t=1}^{T} \frac{\max\left[Z(t),\ 0\right](1+r)^{T-t}}{(1+MIRR)^T}. \tag{3.31}$$

3.5.3 Sharpe ratio

The Sharpe ratio (*SR*) measures the excess return per unit of standard deviation for an investment. Therefore, it characterizes how well the return of an investment compensates the investor for the risk taken. The ratio is calculated by

$$SR = \frac{\frac{1}{T}\sum_{t=1}^{T} \mathrm{Re}(t) - r}{\sigma}, \tag{3.32}$$

where σ is the empirical standard deviation of the investment return, and the return is defined by $\mathrm{Re}(t) = \frac{W(t) - W(t-1)}{W(t-1)}$ for t =1,……,*T*. An investment with a higher *SR* offers a higher return for the same risk.

3.5.4 Insolvency ratio

An insurer's insolvency risk becomes proportionally larger as the liability increases and the equity decreases. To measure the insolvency risk caused by large catastrophes, we define the insolvency ratio $ISR(t)$ as the liability divided by the equity:

$$ISR(t) = L(t) / E(t), \tag{3.33}$$

where a larger insolvency ratio means that there is a higher chance of the insurer defaulting.

4. The calibration of loss model

The source of the insured losses caused by storms is from the publication “Sigma” issued by Swiss Re every year. The journal regularly reports all types of catastrophe losses that have occurred in the previous year. This study collects the data of the storm events for which the insured claims exceeded USD 48.8 million from 2002 to 2014, with 275 observations in North America and 46 in Europe. The loss data are adjusted to the 2014 price level by using the US consumer price index. If the insured loss is estimated with a range, we take the middle value as the insured loss for the event. Tables 1a and 1b display the descriptive statistics of the insured storm losses (in millions of USD, based on the 2014 price level) in North America and Europe, respectively. Table 1a shows that the losses in North America vary from a minimum of 51.56 to a maximum of 8,419.56 (million USD). The standard deviation of 946.79 is much larger than the mean of 584.68. The data distribution has more right skewness of 5.15 and much higher kurtosis of 35.88 than the normal distribution. The augmented Dickey-Fuller (ADF) test statistic reflects that the time series loss data are stationary. The data in Table 1b also show a similar result in Europe as that in North America. Figure 2a and Figure 2b plot the sample autocorrelation of the time series loss data with 95% conference intervals. One autocorrelation signature at a lag of 3 with a sample autocorrelation of 0.2606 in Figure 2a displays a serially dependent characteristic, and one autocorrelation signature at a lag of 9 with a sample autocorrelation of 0.3497 in Figure 2b does as well. Consequently, we need to find an insured storm loss model to satisfactorily describe the statistic characteristics of the loss data mentioned above.

[Insert Table 1a and Table 1b here]

[Insert Figure 2a and Figure 2b here]

The empirical results in Tables 2a and 2b show that the CIR model (as shown in Appendix) is the best-fitting model for the insured storm losses in North America and Europe. The empirical experiments on the insured storm losses in North America and Europe are conducted by the maximum likelihood estimation for the distributions and models presented in the previous literature, such as the lognormal, gamma, generalized Pareto, generalized extreme value, exponential distributions, point process, and the CIR model. The estimation results of the CIR model in Tables 2a and 2b show that it can offer the maximum log-likelihood (-7.34 for North America and -7.21 for Europe) and the minimum AIC (20.69 for North America and 20.43 for Europe) and BIC (31.54 for North America and 25.91 for Europe).

[Insert Table 2a and Table 2b here]

5. Simulation results and analysis

In this section, we estimate the expected value of the performance measures and the different levels of insolvency ratios in asset, liability and equity hedging against the NatCat risk. The Monte Carlo estimates are based on 100,000 independent replicates calculated by the loss model over a period of 30 years. The parameters of the simulation base are defined in Table 3, referring to the studies of Dacorogna et al. (2013) and Wu (2015). The 95% asymptotic confidence interval for the expected values of the performance measures is calculated using these simulation results.

[Insert Table 3 here]

5.1 Financial performance without hedging against the NatCat risk under different maximum claims

Consider an insurer who does not adopt any hedging method, and shareholders raise all *RACs* for issuing new NatCat insurance policies. Thus, the initial equity and liability are *RAC* and zero, respectively. Table 4 shows the insurer's insolvency profile and shareholders' expected performance measures (*PI*, *MIRR*, and *SR*) under different maximum claim levels. The positive expected values of *PI*, *MIRR*, and *SR* ensure that the long-term profit exists based on the insured losses following the CIR model. These three expected values are not significantly different under different maximum claim levels. This means that although the insurance premium income becomes greater as the maximum claim becomes larger, the claim also becomes larger. The larger maximum claim offers the insurer less protection against the NatCat risk. The insolvency profile in Table 4 reflects the occurrence frequency of financial distress for the insurer within 30 years. For instance, the insurer will face approximately one instance of financial distress over the period of 30 years for *ISR* >1.

[Insert Table 4 here]

5.2 Financial performance with asset hedging against the NatCat risk under different maximum claims

Consider an insurer who adopts the asset hedging against the NatCat risk, such as catastrophe derivatives with NatCat-related underlying assets or indexes. Assume that 0.8*RAC comes from its shareholders, and subsequently the insurer buys the catastrophe derivatives to provide the remainder (0.2*RAC) of the risk-adjusted capital. The contingent capital of 0.2*RAC would be used to cover the claim loss over the predetermined strike price ($M(t) – N(t)$). Therefore, the initial equity and liability are 0.8*RAC and nil, respectively. Table 5 shows the insurer's insolvency profile and shareholders' expected performance measures (*PI*, *MIRR*, and *SR*) under different maximum claim levels. The positive expected values of *PI*, *MIRR*, and *SR* ensure that the long-term profit exists based on the insured losses following the CIR model. It shows the feasibility that an insurer can make up for the deficiency of *RAC* by using catastrophe derivatives for the NatCat insurance policies. Over the period of 30 years, the occurrence frequency of financial distress for insolvency ratio > 1 in Table 5 gradually becomes lower than that in Table 4 as the maximum claim level decreases. The occurrence frequency of serious financial distress for *ISR* > 2 becomes significantly smaller as the maximum claim decreases (from 1**RAC* to 0.8**RAC*). Based on the equivalent capital requirement, the occurrence frequency of financial distress for 0.8**RAC* in Table 5 is less than that for the corresponding 1**RAC* in Table 4. It shows that catastrophe derivatives can be used to reduce the insolvency risk.

[Insert Table 5 here]

5.3 Financial performance with liability hedging against the NatCat risk under different maximum claims

Consider that an insurer adopts the liability hedging against the NatCat risk, such as CatBonds with NatCat-related underlying assets or indexes. Assume that 0.8**RAC* comes from its shareholders, and the remaining 0.2**RAC* is prepared by issuing CatBonds, The payment on CatBonds will be triggered under some predetermined conditions. Thus, the initial equity and liability are 0.8**RAC* and nil, respectively. The standby equity of 0.2**RAC* would be used to return to the bondholders or compensate the claimers for the loss at the end of the issue year. Table 6 shows the insurer's insolvency profile and the shareholders' expected performance measures (*PI*, *MIRR*, and *SR*) under different maximum claim levels. The positive expected values of *PI*, *MIRR*, and *SR* ensure that the long-term profit exists based on the insured losses following the CIR model. It shows the feasibility that an insurer can make up for the deficiency of *RAC* by issuing CatBonds for the NatCat insurance policies. Over the period of 30 years, the occurrence frequency of financial distress (for insolvency ratio

> 1) in Table 6 gradually becomes lower than that in Table 4 as the maximum claim level decreases. The occurrence frequency of serious financial distress for *ISR* > 2 rapidly becomes smaller as the maximum claim decreases. Based on the equivalent capital requirement, the occurrence frequency of financial distress for 0.8**RAC* in Table 6 is less than that for the corresponding 1**RAC* in Table 4. It shows that CatBonds can also be used to reduce the insolvency risk.

[Insert Table 6 here]

5.4 Financial performance with equity hedging against the NatCat risk under different maximum claims

Consider that an insurer adopts the equity hedging against the NatCat risk, such as CatEPuts with NatCat-related underlying assets or indexes. Assume that 0.8**RAC* comes from its shareholders, and the remaining 0.2**RAC* is raised by purchasing and exercising the CatEPuts. Thus, the initial equity and liability are 0.8**RAC* and zero, respectively. The standby equity of 0.2**RAC* would be used to compensate the claimers for the loss at the end of the issue year. If the severe impact of catastrophe events occurs frequently, the original shareholders' equity would be diluted by the insurer issuing more equity to the seller of the CatEPuts. Table 7 shows the insurer's insolvency profile and the shareholders' expected performance measures (*PI*, *MIRR*, and *SR*) under different maximum claim levels. The positive expected values of *PI*, *MIRR*, and *SR* ensure that the long-term profit exists based on the insured losses following the CIR model. It shows the feasibility that an insurer can make up for the deficiency of *RAC* by purchasing CatEPuts for the NatCat insurance policies. Although the expected values of *PI* in Table 7 are greater than those in Tables 4, 5, and 6, the expected values of *MIRR* and *SR* are less than those in Tables 4, 5, and 6. The hedging performance of the CatEPuts is not necessarily better than that of the catastrophe derivatives and

CatBonds. Over the period of 30 years, the occurrence frequency of financial distress for insolvency ratio > 1 gradually becomes lower than that in Table 4 as the maximum claim level decreases. The instances of serious financial distress for *ISR* > 2 rapidly becomes smaller as the maximum claim decreases. Based on the equivalent capital requirement, the occurrence frequency of financial distress for 0.8**RAC* in Table 7 is less than that for the corresponding 1**RAC* in Table 4. This shows that CatEPuts can also be used to reduce the insolvency risk.

[Insert Table 7 here]

5.5 Analysis of the different premium payment terms

Stable premium payments are good for the insurer and the insured; however, the premium must be adjusted by the loss ratio frequently. The numerical analysis is designed to find a payment term such that the standard deviation of the premiums within 30 years is the minimum. We consider five different insurance periods of 1, 2, 3, 4, and 5 years under the annual premium for the multi-year insurance and the rolling-year insurance policies. Table 8 shows that the means of the premium volatilities over the period of 30 years decreases as the duration increases, but the standard deviation of the premium volatilities over the period of 30 years is minimized for the insurance period of 3 years. Thus, the payment term of three years is the appropriate period of the stable premium for the insurance policies against the catastrophe risk of winter storms.

[Insert Table 8 here]

6. Conclusions

Out of the natural catastrophes in North America and Europe, the winter storms are primarily responsible for most of the insured losses, and the insured storm losses present a serial dependence. According to the statistical characteristics, the CIR model for storm loss claims is shown to be the better-fitting model among the models studied in the prior literature. Based on this model, we can estimate the risk-adjusted capital and the technical insurance premium every year and set up the insurer's accounting procedures. The numerical analysis results of the financial performance show that the asset (catastrophe derivatives), liability (catastrophe bonds), and equity (catastrophe equity puts) hedging strategies are effective in view of both the long-term positive profit and the improvement in the insolvency ratios (liability divided by equity). Every method or instrument against the NatCat risk simultaneously provides some advantages and disadvantages. Each insurer has its own idiosyncratic capital structure and deals with its local insurance and capital markets. The recommendations of the study provide the insurers with a more diversified portfolio under the catastrophe risk management; however, no single hedging instrument consistently dominates the others in terms of financial performance.

Two feasible methods for premium payment are presented to reduce the volatility of the annual insurance premiums. As the losses of catastrophic disasters vary widely over time, the insurance premiums become extremely volatile. The stable premiums benefit both the insurer and the insured for several years. One method is for the insured to purchase the n-year insurance from the insurer one time with the premium amortized within n years, i.e., the annual premium is the same during the n years. The premium will be adjusted every n years. The other method is for the insured to buy the n-year insurance but with only $1/n$ share of the original maximum claim every year. Then, the insurance coverage will be almost invariable because there are always n policies to cover the NatCat risk every year. The numerical analysis results of the different terms show that the coverage term of $n = 3$ lessens the volatility of the premiums.

Appendix

The CIR model is in the form of $dX(t) = a(b - X(t))dt + \sigma_X \sqrt{X(t)} dW(t)$, where $X(t)$ stands for the insured storm losses; a, b, and σ_X denote the mean-reverting speed, mean, and instantaneous volatility rate, respectively; W follows the Wiener process. Furthermore, $2cX(t)$ follows a non-central chi-square χ^2 distribution with $2q + 2$ degrees of freedom and non-centrality parameter $2u$, where $c = \frac{2a}{\sigma_X(1-e^{-a})}$, $u = cr(t-1)e^{-a}$, $q = \frac{2ab}{\sigma_X} - 1$. The maximum likelihood estimation in this study is based on a true CIR distribution. The following steps are implemented: The first step is to derive the log-likelihood function of CIR model. The second step is to estimate the initial points to find the global optimum. We get the initial parameter estimates in North America (Europe) from 275 (46) observations by using the Ordinary Least Square (OLS) regression on the discretized version of the CIR model, $X(t + \Delta t) - X(t)$ $= a(b - X(t))\Delta t + \sigma_X \sqrt{X(t)}\varepsilon(t)$, where $\varepsilon(t)$ is normally distributed with mean zero and variance Δt. The third step is to achieve the maximum of the log-likelihood function. According to the parameters estimated in Table 2a, without loss of generality, we estimate the claim $2cX(0)$ in year 0 by using a = 4.1170, b = 585.8461, σ_X = 71.4249, and $X(0)$ = 357.403.

References

Abdessalem, M. B., Ohnishi, M., 2014. Catastrophe risk derivatives: a new approach. *Journal of Mathematical Finance*, 4 (01), 21-34. doi:10.4236/jmf.2014.41003.

Burnecki, K., Kukla, G., Weron, R., 2000. Property insurance loss distributions. *Physica A: Statistical Mechanics and its Applications*, 287(1-2), 269-278. doi:10.1016/S0378-4371(00)00453-2.

Christensen, C. V., 1999, A new model for pricing catastrophe insurance derivatives. *CAF Working Paper* 28, Denmark: University of Aarhus.

Cox, J. C., Ingersoll, J. E., Ross, S. A., 1985. A theory of the term structure of interest rates. *Econometrica*, 53 (2), 385-407. doi:10.2307/1911242.

Dacorogna, M. M., Albrecher, H., Moller, M., Sahiti, S., 2013. Equalization reserves for natural catastrophes and shareholder value: a simulation study. *European Actuarial Journal*, 3 (1), 1-22. doi:10.1007/s13385-013-0062-z.

Jaimungal, S., Wang, T., 2006. Catastrophe options with stochastic interest rates and compound Poisson losses. *Insurance: Mathematics and Economics*, 38 (3), 469-483. doi:10.1016/j.insmatheco.2005.11.008.

Kladivko, K., 2007. Maximum likelihood estimation of the Cox-Ingersoll-Ross process: the MATLAB implementation. *Technical Computing Prague*, working paper.

Lin, S., Chang, C., Powers, M.R., 2009. The valuation of contingent capital with catastrophe risks. *Insurance: Mathematics and Economics*, 45 (1), 65-73. doi:10.1016/j. insmatheco. 2009.03.005.

Pérez-González, F., Yun, H., 2013. Risk management and firm value: evidence from weather derivatives. *Journal of Finance*, 68 (5), 2143-2176. doi:10.1111/jofi.12061.

Powers, M.R., Powers, T. Y., Gao, S., 2012. Risk finance for catastrophe losses with Pareto-calibrated Levy-stable severities. *Risk Analysis*, 32 (11), 1967-1977.

doi:10.1111/j.1539-6924.2012.01906.x.

Smith, R. L., Shively, T. S., 1995. Point process approach to modeling trends in tropospheric ozone based on exceedances of a high threshold. *Atmospheric Environment*, 29 (23), 3489-3499. doi:10.1016/1352-2310(95)00030-3.

Vaugirard, V. E., 2003. Pricing catastrophe bonds by an arbitrage approach. *Quarterly Review of Economics and Finance*, 43 (1), 119-132. doi:10.1016/S1062-9769(02)00158-8.

Von Dahlen, S., Von, G., 2012. Natural catastrophes and global reinsurance − exploring the linkages. *BIS Quarterly Review*. 1111.

Wu, Y., 2015. Reexamining the feasibility of diversification and transfer instruments on smoothing catastrophe risk. *Insurance: Mathematics and Economics*, 64, 54-66. doi:10.1016/j.insmatheco.2015.04.007.

Wu, Y., Chung, S., 2010. Catastrophe risk management with counterparty risk using alternative instruments. *Insurance: Mathematics and Economics*, 47 (2), 234-245. doi:10.1016/j.insmatheco.2010.04.002.

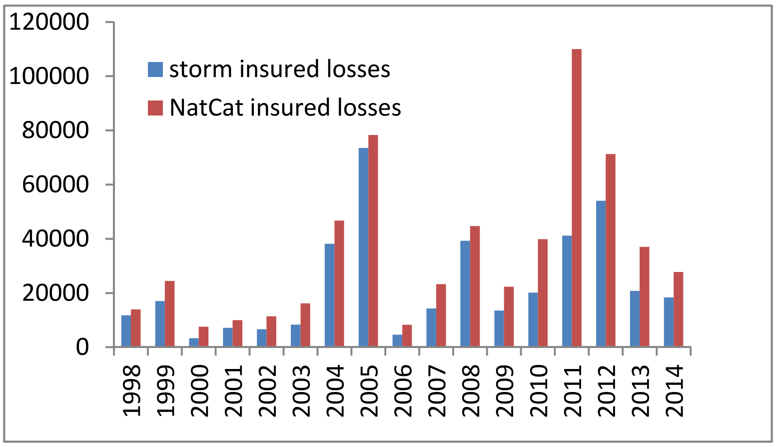

Figure 1 NatCat insured losses and storm insured losses

(in millions of USD based on 2014 price level)

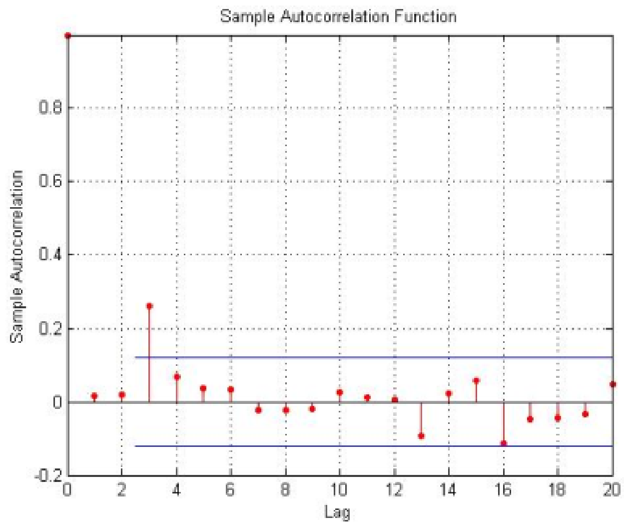

Figure 2a The autocorrelation plot for the storm losses in North America

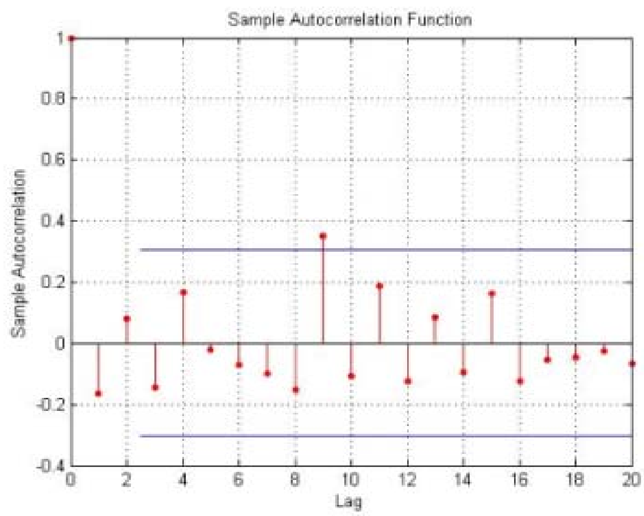

Figure 2b The autocorrelation plot for the storm losses in Europe

Table 1a. Descriptive statistics of the insured storm losses in North America from 2002 to 2014 in millions of USD.

	N	Mean	Median	S.D.	Min	Max	Skew	Kurt.	J-B(*p*)	ADF(*p*)
Insured losses	275	584.68	228.35	946.79	51.56	8,419.56	5.15	35.88	15408(0.001)	-12.30(0.001)

Notes: 1. Source: Sigma from Swiss Re 2002 to 2014, in 2014 price level.

2. This table shows the descriptive statistics for the insured losses. The number of observations (*N*), the mean, the median, the standard deviation (S.D.), the minimum (Min), the maximum (Max), and the coefficients of skewness (Skew) and Kurtosis (Kurt) are reported. J-B denotes the test statistic for the Jarque-Bera normality test, which has a chi-square distribution with two degrees of freedom. *p* denotes the *p*-value in parentheses with the value 0.001 represents a p-value less than 0.001. ADF represents the Augmented Dickey-Fuller test statistic.

Table 1b. Descriptive statistics of the insured storm losses in Europe from 2002 to 2014 in millions of USD.

	N	Mean	Median	S.D.	Min	Max	Skew	Kurt.	J-B(*p*)	ADF(*p*)
Insured losses	46	605.50	239.50	1,065.33	5.00	6,100.00	3.66	15.58	461.77(0.001)	-5.86(0.001)

Notes: 1. Source: Sigma from Swiss Re 2002 to 2014, in 2014 price level.

2. This table shows the descriptive statistics for the insured losses. The number of observations (*N*), the mean, the median, the standard deviation (S.D.), the minimum (Min), the maximum (Max), and the coefficients of skewness (Skew) and Kurtosis (Kurt) are reported. J-B denotes the test statistic for the Jarque-Bera normality test, which has a chi-square distribution with two degrees of freedom. *p* denotes the *p*-value in parentheses with the value 0.001 represents a p-value less than 0.001. ADF represents the Augmented Dickey-Fuller test statistic.

Table 2a. Maximum likelihood estimation of the insured storm losses in North America

Estimated distribution parameters	Lognormal	Gamma	Generalized Pareto	Generalized extreme value	Exponential	Point Process	CIR
Scale	1.0313	617.2137	407.6014	179.0435	585.8423	2,188.5650	
Shape		0.9473	0.2990	0.7803		0.2825	
Location	5.7585			204.3687		6,285.9438	
Mean-reverting speed							4.1170
Mean							585.8461
Instantaneous volatility rate							71.4249
Log-Likelihood	-1975.04	-2078.90	-2000.09	-1968.94	-2020.22	-657.39	-7.34
AIC	3954.09	4161.80	4004.18	3943.92	4042.43	1320.78	20.69
BIC	3961.31	4169.03	4011.41	3954.76	4046.04	1331.61	31.54

Notes: 1. AIC denotes the Akaike Information Criterion for the estimated model.
2. BIC denotes Bayesian Information Criterion
3. CIR model denotes the Cox-Ingersoll-Ross mean-reverting square root models.

Table 2b. Maximum likelihood estimation of the insured storm losses in Europe

Estimated distribution parameters	Lognormal	Gamma	Generalized Pareto	Generalized extreme value	Exponential	Point process	CIR
Scale	1.3047	873.4036	296.7604	170.8691	605.4956	6,012.3861	
Shape		0.6933	0.5358	0.8252		0.5098	
Location	5.5331			160.7423		11,209.4898	
Mean-reverting speed							16.3962
Mean							616.7786
Instantaneous volatility rate							170.6214
Log-Likelihood	-332.03	-345.98	-332.99	-331.13	-340.68	-107.57	-7.21
AIC	668.06	695.97	669.98	668.26	683.36	221.14	20.43
BIC	671.72	699.63	673.63	673.75	685.19	226.63	25.91

Notes: 1. AIC denotes the Akaike Information Criterion for the estimated model.
2. BIC denotes Bayesian Information Criterion
3. CIR model denotes the Cox-Ingersoll-Ross mean-reverting square root models.

Table 3. Standard set of parameters

Standard parameters	
Initial claim in year 0	83.7691 million USD
Claim coverage rate θ	0.99
Risk-free rate r	2%
Loan rate for liability c	3%
Shareholder's required return κ	15%
Expense e	1% of expected loss
Tax rate τ	25%
Tax shield rate γ	25%
Life of the investment T	30
Simulation times	100,000
Predetermined ratio of RAC(t) for CatBonds, CatEPuts, and derivatives h	0.2
Discount rate of CatBonds ρ	0.9
Initial shares outstanding n_1	10
New shares issued for the exercise of CatEPuts	1

Notes: 1. The parameters of the simulation base refer to the studies of Dacorogna et al. (2013) and Wu (2015).

2. The initial claim in year 0 is estimated by the methods described in the Appendix.

Table 4. Financial performance without hedging against the NatCat risk under different maximum claims

Maximum claim level		1**RAC*	0.90**RAC*	0.8**RAC*
E(*PI*)		0.7590	0.7150	0.6587
E(*MIRR*)		0.1100	0.1096	0.1090
E(*SR*)		0.3861	0.3859	0.3836
insolvency ratio	> 0.5	2.0402	2.0272	2.0305
insolvency ratio	> 1	0.9672	0.9550	0.8365
insolvency ratio	> 1.5	0.5878	0.5318	0.1761
insolvency ratio	> 2	0.4131	0.2316	0.0895

Notes: The numbers listed for different insolvency ratios denote the occurrence frequency of financial distress within 30 years.

Table 5. Financial performance with catastrophe derivatives hedging against the NatCat risk under different maximum claims

Maximum claim level		1**RAC*	0.90**RAC*	0.8**RAC*
E(*PI*)		0.6028	0.4631	0.3308
E(*MIRR*)		0.1028	0.1017	0.1006
E(*SR*)		0.3183	0.3064	0.3044
insolvency ratio	> 0.5	3.1072	2.9665	2.9095
insolvency ratio	> 1	1.5453	1.5154	0.7543
insolvency ratio	> 1.5	1.042	0.6364	0.1612
insolvency ratio	> 2	0.6023	0.2002	0.0853

Notes: The numbers listed for different insolvency ratios denote the occurrence frequency of financial distress within 30 years.

Table 6. Financial performance with CatBonds hedging against the NatCat risk under different maximum claims

Maximum claim level		1*RAC	0.90*RAC	0.8*RAC
E(PI)		0.5142	0.5554	0.6035
E(MIRR)		0.1024	0.1025	0.1027
E(SR)		0.3115	0.3147	0.3164
insolvency ratio	> 0.5	2.9698	2.9302	2.7554
insolvency ratio	> 1	1.5808	1.5103	0.5543
insolvency ratio	> 1.5	1.0681	0.6258	0.1130
insolvency ratio	> 2	0.6952	0.2034	0.0621

Notes: The numbers listed for different insolvency ratios denote the occurrence frequency of financial distress within 30 years

Table 7. Financial performance with CatEPuts hedging against the NatCat risk under different maximum claims

Maximum claim level		1*RAC	0.90*RAC	0.8*RAC
E(PI)		1.1661	1.2188	1.2588
E($MIRR$)		0.0001	0.0001	0.0001
E(SR)		0.0004	0.0004	0.0003
insolvency ratio rate	> 0.5	2.0373	1.8294	0.9325
insolvency ratio rate	> 1	0.7584	0.2253	0.0242
insolvency ratio rate	> 1.5	0.2256	0.0383	0.0051
insolvency ratio rate	> 2	0.0986	0.0124	0.0053

Notes: The numbers listed for different insolvency ratios denote the occurrence frequency of financial distress within 30 years.

Table 8. The standard deviation of premium payments for different terms and methods

Period of insurance coverage	Technical premium	Mean*	Standard deviation**
1	TP	0.9604	0.0170
2	TP_2	0.8345	0.0110
	TP_2^r	0.8271	0.0109
3	TP_3	0.6615	0.0087
	TP_3^r	0.6561	0.0088
4	TP_4	0.5720	0.0178
	TP_4^r	0.5737	0.0174
5	TP_5	0.4544	0.1599
	TP_5^r	0.4472	0.1573

Notes: 1. TP_n denotes the yearly premium estimated by amortizing the *n*-year premium.

2. TP_n^r denotes the yearly premium for the *n*-year policy with 1/*n* share of the maximum claim.

3. * represents the mean of 100,000 independent replicates for means of premium volatilities within 30 years.

4. ** represents the standard deviation of 100,000 independent replicates for standard deviations of premium volatilities within 30 years.